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EFFECT OF TANGENTIAL FORCES OF INERTIA ON THE FREQUENCY OF FREE OSCILLATIONS IN A THIN CYLINDRICAL SHELL

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EFFECT OF TANGENTIAL FORCES OF INERTIA ON THE FREQUENCY OF FREE OSCILLATIONS IN A THIN CYLINDRICAL SHELL*

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Estimate of the error arising when the inertial forces due to tangential displacements are not taken into account in determining free oscillation frequencies of thin, circular, cylindrcial shells. The error arising when r/ℓ is equal to, greater than, or smaller than 0.5 is discussed. Author

This note presents an estimate of the error arising when the inertial forces due to tangential displacements u and v are not taken into account in determining the free oscillation frequencies of thin, circular, cylindrical shells. /212

If ξ and θ are the coordinates of the mean surface of the shell, and u, v and w are displacements directed along ξ, θ and the normal, respectively, then the problem of the free oscillations of a circular cylindrical shell reduces to the integration of the following system of differential equations:

$$\begin{aligned}
&\frac{\partial^2 u}{\partial \xi^2}+\frac{1-\nu}{2}\frac{\partial^2 u}{\partial \theta^2}+\frac{1+\nu}{2}\frac{\partial^2 v}{\partial \xi \partial \theta}+\nu\frac{\partial w}{\partial \xi}=\frac{1-\nu^2}{gE}r^2\gamma\frac{\partial^2 u}{\partial t^2},\\
&\frac{\partial^2 v}{\partial \theta^2}+\frac{1-\nu}{2}\frac{\partial^2 v}{\partial \xi^2}+\frac{1+\nu}{2}\frac{\partial^2 u}{\partial \xi \partial \theta}+\frac{\partial w}{\partial \theta}=\frac{1-\nu^2}{gE}r^2\gamma\frac{\partial^2 v}{\partial t^2},\\
&\frac{1}{k^4}\Delta\Delta w+w+\nu\frac{\partial u}{\partial \xi}+\frac{\partial v}{\partial \theta}=-\frac{1-\nu^2}{gE}r^2\gamma\frac{\partial^2 w}{\partial t^2},
\end{aligned} \tag{1}$$

where

$$k^4=\frac{12r^2}{h^2}. \tag{2}$$

Since an exact solution of this problem exists only for a shell suspended by hinges on its faces, this case will also be studied.

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/Numbers in the margin indicate pagination in the original foreign text.

Thus, on the faces of the shell, $\xi = \theta$ and $\xi_0 = \frac{\ell}{r}$

$$v = w = T_1 \quad и \quad M_1 = 0. \tag{3}$$

Under the conditions of (3), equations (1) are satisfied by the solution

$$\begin{aligned} u &= a \cos \alpha_m \xi \sin n\theta \sin \omega t, \\ v &= b \sin \alpha_m \xi \cos n\theta \sin \omega t, \\ w &= c \sin \alpha_m \xi \sin n\theta \sin \omega t, \end{aligned} \tag{4}$$

where n is the number of waves formed along the perimeter of the shell, ω is /213 the angular frequency, $\alpha_m = \frac{m\pi r}{\ell}$, and m is the number of half-waves forming along the shell.

The further study of this problem is made more convenient by excluding from system (1) the tangential displacements u and v, thus reducing the problem to the study of a single differential equation of the eighth order with respect to the normal displacement w:

$$\begin{gathered} \frac{1}{k^4}\Delta\Delta\Delta\Delta w + (1-\nu^2)\frac{\partial^4 w}{\partial \xi^4} = -\frac{2}{1+\nu}\frac{r^2\gamma}{gE}\frac{\partial^2}{\partial t^2} \times \\ \times \left\{ \frac{1-\nu^2}{gE} r^2\gamma \frac{\partial^2}{\partial t^2}\left[\frac{1-\nu^2}{gE} r^2\gamma \frac{\partial^2 w}{\partial t^2} + \right.\right. \\ \left. + \frac{\Delta\Delta w}{k^4} + w\right] - \frac{3-\nu}{2}\Delta\left[\frac{1-\nu^2}{gE} r^2\gamma \frac{\partial^2 w}{\partial t^2} + \frac{\Delta\Delta w}{k^4} + w\right] + \\ \left. + \nu^2 \frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \theta^2} + \frac{1-\nu}{2}\Delta\Delta w \right\}. \end{gathered} \tag{5}$$

If the tangential forces of inertia are excluded in system (1), then instead of equation (5) one obtains

$$\frac{1}{k^4}\Delta\Delta\Delta\Delta w + (1-\nu^2)\frac{\partial^4 w}{\partial \xi^4} = \frac{1-\nu^2}{Eg} r^2\gamma \frac{\partial^2}{\partial t^2}(\Delta\Delta w). \tag{6}$$

Substituting solution (4) into equation (6) yields the following value for the frequency:

$$\omega^2_{n,m} = \frac{Eh^2 g}{12(1-\nu^2)r^4\gamma}\left(\alpha_m^2 + w^2\right)^2 + \frac{Eg}{r^2\gamma}\frac{\alpha_m^4}{\left(\alpha_m^2 + n^2\right)^2}. \tag{7}$$

If the same solution (4) is substituted into equation (5), in which the tangential forces of inertia are considered, one obtains the following equation for the frequency:

$$Z^3 - AZ^2 + BZ - C = 0, \tag{8}$$

in which

$$Z = \frac{1-\nu^2}{gE} r^2 \gamma \bar{\omega}^2_{m,n},$$
$$A = (\alpha_m^2 + n^2)\left(\frac{\alpha_m^2 + n^2}{k^4} + \frac{3-\nu}{2}\right); \tag{9}$$

$$B = \frac{1-\nu}{2}(\alpha_m^2 + n^2)^2 \left[1 + \frac{3-\nu}{1-\nu}\frac{\alpha_m^2 + n^2}{k^4} + \frac{n^2 + (3+2\nu)\alpha_m^2}{(\alpha_m^2 + n^2)^2}\right].$$
$$C = \frac{1-\nu}{2}(\alpha_m^2 + n^2)^2 \left[\frac{(\alpha_m^2 + n^2)^2}{k^4} + \frac{(1-\nu^2)\alpha_m^4}{(\alpha_m^2 + n^2)^2}\right]. \tag{10}$$

Equation (8) has three real roots.

Numerical solutions of equation (8) for a broad range of relative dimensions of the shell indicate that one of the roots of this equation is significantly smaller than the other two.

Two roots will have values on the order of $1 + (\alpha_m^2 + n^2)$, while the third will be on the order of $\frac{\alpha_m^4}{(\alpha_m^2 + n^2)^2}$. /214

The physical significance of this finding is that the minimal frequency will correspond to oscillations of the shell having the dominant value of the normal displacements.

An approximate method for the solution of the cubic equation (8) follows from what has been said above.

The smallest root of this equation will be given by

$$Z_{\min} = \frac{C}{B}, \tag{11}$$

while the remaining two roots will be given by the equation

$$Z^2 - AZ + B = 0. \tag{12}$$

The error of this approximate method for determining the roots of equation (8) does not exceed 2%.

It should be noted that this question was previously studied by A. S. Solomenko, who also obtained an equation having the form of (8). Because of the complexity of finding accurate values of the roots of this equation, however, he was unable to carry the study to a successful conclusion.

Thus, considering equation (7) and definitions (9), one obtains

$$\bar{\omega}^2_{m,n} = \frac{\omega^2_{m,n}}{1+\Delta}, \tag{13}$$

where

$$\Delta = \frac{3-\nu}{1-\nu}\,\frac{\alpha_m^2+n^2}{k^4} + \frac{n^2+(3+2\nu)\alpha_m^2}{\left(\alpha_m^2+n^2\right)^2}. \tag{14}$$

When $\Delta = 0$, equation (13) becomes equation (7); consequently, the effect of the tangential forces of inertia on the free oscillation frequency of the shell is determined by the value of the parameter Δ.

The graphs shown in Figure 1, 2 and 3 were constructed on the basis of the results of numerical calculations in which the parameter Δ was determined in relation to the tone of the oscillation (m), the number of waves (n) and the ratio $\frac{r}{\ell}$ when the ratio $\frac{h}{r}$ was held constant.

It is obvious from an examination of these graphs that for long shells, characterized by values of the ratio $\frac{r}{\ell} < 0.5$, at the first tone ($m = 1$) the parameter $\Delta \approx \frac{1}{3}$, i.e., it contributes approximately 25% of the error during the determination of the frequency (error in the dangerous direction).

For shells with a ratio $\frac{r}{\ell} \geqslant 0.5$, the effect of the tangential forces of inertia becomes significant only at high tones. Thus, for example, for the first tone, neglecting these forces during determination of the frequency leads to an error on the order of 2%, so that in this case they may safely be neglected. In the case of higher tones, however, neglecting these forces may lead to gross error. Thus, for the fifth tone (m = 5), this error is on the order of 25%. /215

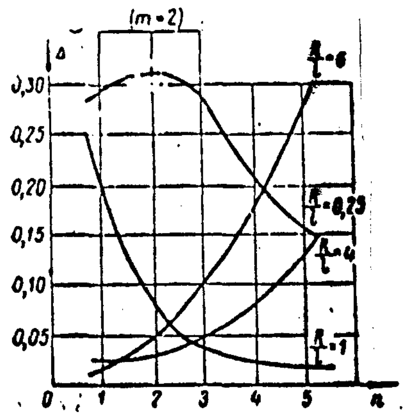

Figure 1

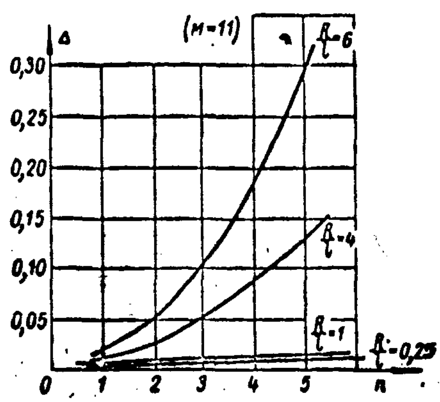

Figure 2

Translator's note: All commas in the numbers appearing in Figures 1, 2 and 3 are to be read as decimal points.

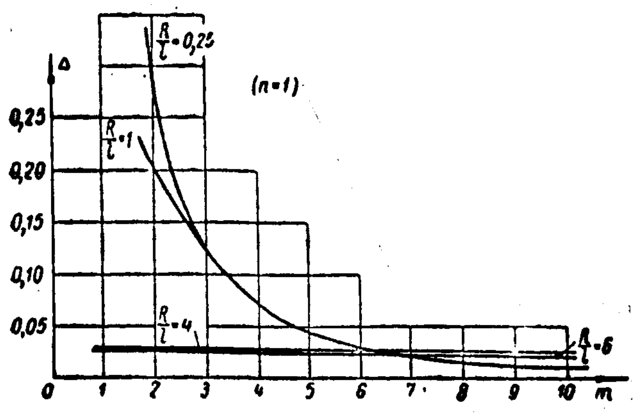

Figure 3

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